Math 3B **Quiz 1**

Name:______________________________ Section Time:__________

Complete the following problems, making sure to SHOW ALL WORK. If you're stuck on something, CLEARLY EXPLAINING what you do know or what you would do will get you partial credit!

1. Consider the function

$$f''(x) = \frac{4x^2 - 2\sqrt{x}}{x^2}.$$

(a) Find the most general form for a function f such that $\frac{d^2 f}{dx^2} = f''$.

(b) Now suppose $f(1) = 5$ and $f'(4) = 18$. Find the particular function f such that $\frac{d^2 f}{dx^2} = f''$.

(a) We begin by finding the first antiderivative. This is straightforward once the fraction has been split apart:

$$f''(x) = \frac{4x^2}{x^2} - \frac{2\sqrt{x}}{x^2} = 4 - 2x^{-3/2}. \qquad (x \neq 0)$$

The antiderivative of this is found simply by reversing the power rule:

$$f'(x) = 4x + 4x^{-1/2} + C.$$

One more antiderivative gives us

$$f(x) = 2x^2 + 8\sqrt{x} + Cx + D.$$

(b) We did most of the work in the previous part. We just need to find the correct values of C and D. Let's begin with the second piece of information we are given. If $f'(4) = 18$, then

$$f'(4) = 4(4) + 4\left(\frac{1}{2}\right) + C = 18,$$

or $C = 0$. Using this and the fact that $f(1) = 5$, we see

$$f(1) = 2(1)^2 + 8(1) + C(0) + D = 5,$$

or $D = -5$. We conclude, then, that the function we set off in search of is

$$f(x) = 2x^2 + 8\sqrt{x} - 5.$$